Understanding “Sequence of Returns”

The DoubleBucket® Team

The phrase “Sequence of Returns” (which we’ll refer to as SoR), is frequently used when comparing the average return of an investment versus the actual return. However, that is a misnomer as sequencing has nothing to do with that comparison. A more accurate phrase might be Variance of Returns (or VoR) as the actual return depends on the variance of the yearly rates. In contrast, SoR comes into play when money is deposited or withdrawn over time from an account that is invested with a variable return rate (e.g., the stock market). The key points of this paper are as follows:

- Difference between SoR & VoR
- Risks associated with SoR & VoR
- Double-Whammy of SoR Risk on the 4% Rule
- Avoiding SoR Risk
- Optimizing Retirement Spending

Variance of Returns (VoR):

Most investors understand that the actual return of an investment asset (or asset mix) is much more important than the average return. For example, let’s say the yearly returns of the S&P 500 over the past 3 years are 0%, 10%, & 20%. To calculate the average, you simply add up the returns and divide by 3, resulting in 10%. To calculate the actual return, the three numbers are multiplied together, and the Nth root is taken (in this case the cubed root). Actual return is also referred to as Compound Annual Growth Rate (CAGR). Multiplying 1.0, 1.10, & 1.2 results in 1.32 and the cubed root is 1.097, for an actual return of 9.7% (note that one is added to each value and then subtracted off the result to get the correct value). Multiplication is commutative, meaning that the order or sequence of the yearly returns doesn’t matter. What does matter is the variance. Let’s take a simple example with whole numbers to understand why that is the case. Let’s say we have a series of 2 numbers that add up to 20 (e.g., 10,10 or 9,11 or 8,12, etc.). The average of all these series is the same (i.e., 10), but the product is different (9x11 is 99, 8x12 is 96, 7x13 is 91). Note that the more the numbers vary, the smaller the product. In general, more variance in a series of rates will result in more variance between the average and actual rates. In addition, there is one very important thing to realize. The average return is the max, or limit, of the actual rate. The actual return can only be the same as the average if the yearly returns are the same each year (fixed return rate). If there is any variance at all, the actual will always be less than the average.

One example of VoR is in comparing the NASDAQ against the S&P 500 over the last 50 years. We will use the standard deviation to measure the variance. Note the correlation between variance (standard deviation) and difference of the average and actual returns:

1972-2021	S&P 500	NASDAQ
Average Return	12.3	14.6
Actual Return (CAGR)	10.9	11.6
Difference	1.4	3.0
Standard Deviation	16.8	25.0

As we would expect, the standard deviation of the NASDAQ over this period is much greater than the S&P 500. Correspondingly, we see a wider difference between the average and actual returns for the NASDAQ versus the S&P 500.

Reducing Variance of Returns Risk:

As we see above, the average return of the NASDAQ over 50 years was 14.6%, but the actual return was 11.6%. The associated risk is in using the average to estimate future returns. That is why it's important to diversify assets and rebalance on a periodic basis to reduce the risk. This is nothing new. You can't pick up a copy of Money Magazine without this topic being discussed in several articles over the course of a year. However, it's always good to see a concrete example of how this plays out over history. Below, we show the returns of S&P 500 and the NASDAQ over the last 50 years and the returns of a 50/50 split between the 2 (with rebalancing on a yearly basis). Note that the actual return of a 50/50 split is nearly the same as the NASDAQ (11.4 vs. 11.6). What this means is that the risk of investing in the NASDAQ has been reduced by 50%, with almost the same return. In addition, the actual return of the 50/50 split is a half point higher than the S&P 500 (10.9 vs. 11.4). Reducing risk while maintaining high returns is a key benefit of diversification:

1972-2021	S&P 500	NASDAQ	50/50 Split
Average Return	12.3	14.6	13.4
Actual Return	10.9	11.6	11.4

Sequence of Returns (SoR):

Getting back to the title of this paper, let's dive into the details of Sequence of Returns. SoR is a situation where sequencing (ordering) of yearly rates does affect the overall return. This happens when money is deposited or withdrawn during the span. Here are some examples where SoR will affect the outcome:

- Borrowing against a residence to invest elsewhere
- Borrowing against a Margin Account to invest elsewhere
- Periodic contributions to savings
- Periodic withdrawals from savings

The crux of the difference between VoR and SoR is time. We can predict results much better over long time-periods than short time-periods. For VoR, we only had to concern ourselves with return rates over a long timeframe. With SoR, we are still considering long timeframes, but the interim payments and

withdrawals are made over short time periods where we cannot predict the ups and downs of the underlying asset.

Let's go back to our initial VoR example where the yearly returns of the S&P 500 over a 3-year time-period were 0%, 10%, & 20%. For the VoR example, you would start with an initial deposit and the sequence of returns would not affect the end-result. The ordering could be 0,10,20 or 20,10,0 and the result would be the same. However, when you throw in yearly contributions, you start seeing the effects of sequencing. In this example, we will start with zero and add in a yearly deposit of $1000 and see how sequencing affects the results.

Sequence #1: 0, .10, .20:

Year	Balance	Contribution	Total	Return Rate	End Balance
1	0	1000	1000	0	1000
2	1000	1000	2000	10	2200
3	2200	1000	3200	20	3840

Sequence #2: .20, .10, 0:

Year	Balance	Contribution	Total	Return Rate	End Balance
1	0	1000	1000	20	1200
2	1200	1000	2200	10	2420
3	2420	1000	3420	0	3420

The ending balance is over 10% higher for the first sequence in comparison to the second, and this is a short time-period. For longer time periods, the difference can be more dramatic. Unfortunately, there is no mathematical model available for SoR like we saw with VoR where standard deviations effect the actual return. Unless you can predict short term swings in the market, you can't predict the effects of SoR. However, we can look at actual results over history to get a feel for the effects of SoR, which help put strategies in place to reduce risk.

Sequence of Returns (SoR) Risk:

If we look at the effect of SoR on retirement savings, the fundamental risk is that the ending principal is less than the projected amount (using an estimated return). Conversely, the risk with retirement withdrawals is running out of principal. In addition to those risks, you also have the unintended consequences of saving too much or spending too little. Those are not risks, but annoyances in that you could've either spent more in retirement or saved less on the front end. That's the unfortunate thing with SoR in that end-results can vary dramatically. When doing an analysis of the 4% Rule, we find that some 30-year spans come close to running out while others end with multiples of the starting principal. To avoid the risk of saving too little, the usual advice is to save more early in the span and adjust later when the end goal is clearer. Similarly in retirement, spend less early and adjust later. Unfortunately, that's incongruent with reality. Usually, you have less money to save early in your career as opposed to later. In retirement, you'd like to spend more in the early years because of a more active lifestyle. That's not ideal but there's not much we can do about it. To avoid the risk, it all goes back to the mantra

"Save more, spend less". Let's look at some historical spans to see how SoR can affect both retirement savings and retirement spending.

We'll first look at an example where SoR can affect retirement savings. History tells us that the CAGR for a mix of a 70/30 split of stocks and bonds was approximately 10.2% over 30 years starting in both 1973 (1973 thru 2002) and 1983 (1983 thru 2012). This is using the diversification calculator found on the DoubleBucket® website (https://www.thedoublebucket.com/historical-analysis). If you were 25 years old in either 1973 or 1983 and could predict the future, you could do a Time Value of Money (TVM) calculation to see the savings needed to reach a retirement goal of $2m in 30 years. Doing the calculation, we find that contributing $12,000 at the end of each year (~$1000/mo) would arrive at that goal. However, that assumes a consistent 10.2% each year. Instead, if we do the calculation using the exact yearly rates which go up and down each year, we find different results. The span starting in 1973 resulted in savings of roughly $2.6m whereas the 1983 span was about $1.6m. In both scenarios, the CAGR and the savings rate were the same, but the results differed by about $1m. This is all due to SoR. To quantify the risk, the shortfall in savings in the 1983 span is over $400k. To make up for that you would need to increase yearly contributions by about 20%. Of course, that would leave the 1973 span way over plan. That might not be a bad thing but living on beans and rice during your working years and caviar and steak in retirement is not ideal either. Here is a table to summarize the effect of SoR on savings:

Span	CAGR	Yearly Deposit	End Principal
Avg 30-yr span	10.2	$12,000	$2,000,000
1973-2002	10.2	$12,000	$2,600,000
1983-2012	10.2	$12,000	$1,600,000

While this example may be somewhat extreme, our modelling shows that SoR definitely affects savings over time. The more interesting question might be, why was there such a stark difference between these spans? The simple answer to that is that the 1973 span underperformed early in the span and overperformed late. In the 1983 span, it was just the opposite. When considering SoR for savings, you want the higher returns to occur later in the span when you've accumulated more.

Now let's look at how SoR can affect retirement spending. Let's say you retired in 1973 using the 4% rule with an initial $1m in savings invested in a 50/50 mix of stocks and bonds. At the end of 30 years, your account balance would be down to about $500k. That's good in that it lasted the full 30 years, but if you needed it for 35 years, you might run out. In contrast, if you retired in 1974 under the same scenario, your ending balance would be over $3m. That's almost hard to believe that 1 year could make that big of a difference but the numbers don't lie. The reason for this is twofold: 1) inflation was high early in these spans, causing the withdrawals to be higher for the 1973 span, and 2) the market took a hit in 1973, which didn't affect the 1974 span. This is one of the problems with the 4% rule in that it gets hit by a double whammy of SoR risk. Not only is there a sequence of market returns risk, there is also a sequence of inflation rates risk. Here is a table to show the effect of SoR on retirement spending in these spans (note that the $40k withdrawal is inflation adjusted each year):

Span	Starting Principal	Yearly Withdrawal	Ending Principal
1973-2002	$1,000,000	$40,000	$500,000
1974-2003	$1,000,000	$40,000	$3,000,000

Avoiding SoR Risk:

The traditional thinking on how to avoid SoR Risk with respect to retirement spending is to plan for the worst case. This is the primary concept of the Monte Carlo model in that it plays out all possible scenarios to see if you're covered in all cases. However, we've done research that makes us believe that SoR Risk can be avoided. Impossible you say? As you suspect, there is a catch. We do eliminate SoR risk, but we replace it with withdrawal variance risk which is the risk of retirement withdrawals varying during retirement. That is contrary to the thinking of the 4% rule in that withdrawals should be the same each year (inflation adjusted). That may comfort some retirees, but for those that don't need to leave an estate and want to optimize their retirement spending, the DoubleBucket® Method (www.TheDoubleBucket.com) may be of interest. Our research shows that withdrawals can be made at rates of 5-6% in about 90% of historical spans. In the 10% of spans that don't perform so well, the withdrawal rates are still above 4% on average.

In Summary:

- The term "Sequence of Returns" is sometimes mistakenly used when comparing actual versus average return rates. Instead, "Variance of Returns" is a better term. SoR only comes into play when deposits or withdrawals are made during the span.

- SoR Risk is a phenomenon that can greatly affect retirement savings or retirement spending. If not considered, young investors run the risk of under saving. Similarly, retirees run the risk of prematurely running out of memory.

- To counter SoR risk, "save more, spend less" is the usual mantra. However, the DoubleBucket Method is an alternative. If retirees are flexible in their yearly spending, average withdrawals are much higher than the 4% Rule.

Studying SoR & VoR is just one application of the DoubleBucket® Method. We have a software library that allows us to analyze a variety of topics such as asset diversification, college savings, social security, and others. For more information, please go to www.thedoublebucket.com/resources and drop us a line if you have any questions.

Thank you,

-The DoubleBucket® Team

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